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Axion stabilization in type IIB flux compactifications

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ABSTRACT: A scenario for stabilization of axionic moduli fields in the context of type IIB Calabi-Yau flux compactifications is discussed in detail. We consider the case of a Calabi-Yau orientifold with $h_-^{1,1} \neq 0$ which allows for the presence of B_2 and C_2 -moduli. In an attempt to generalize the KKLT and the Large Volume Scenario, we show that these axions can also be stabilized - some already at tree level, and others when we include perturbative α' -corrections to the Kähler potential K and nonperturbative D3-instanton contributions to the superpotential W . At last, we comment on the possible influence of worldsheet instantons on the process of moduli stabilization.

KEYWORDS: Flux compactifications, Superstring Vacua, Nonperturbative Effects.

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1. Introduction

In the past few years there has been great research interest in the field of string phenomenology, dealing with the question of stabilizing moduli fields at desirably high masses (for a comprehensive review see e.g. [1, 2]). This was initiated by the KKLT scenario [3] which suggested a way to obtain stabilized vacua from type IIB string theory building on earlier works such as [4, 5]. Presently, one can find many extensions and improvements of the original idea, the most notable and well established of which is the Large Volume Scenario (LVS) [6, 7]. It builds up on the KKLT solutions by allowing for non-supersymmetric vacua and by including perturbative corrections to the tree-level Kähler potential computed in [8]. Up to now the LVS has passed many consistency checks (e.g. [9]), but there is nevertheless much space for improvement. The stabilization of the Kähler moduli requires manifolds with negative Euler number, as well as non-perturbative effects which appear only if certain conditions are satisfied [10–12]. And it is of course desirable to have a working recipe also for the other cases. The process of uplifting to a Minkowski or de Sitter vacuum also

needs to be understood better because at present it seems that unnatural fine tuning of parameters is necessary.

In this paper we propose another extension, namely the stabilization of moduli fields that arise from the two-form R-R and NS-NS fields in type IIB string theory. These are usually neglected in the literature, where the main focus is on stabilizing the volume of the underlying manifold to large enough values. Here we will argue that the stabilization of these so called axionic or non-geometric moduli is an important step in drawing the full picture. We show that these axions may lead to changes in the process of stabilization of the manifold volume and the other moduli. Additional motivation for considering them are the possible cosmological consequences from their existence - they are good candidates for driving inflation as recently suggested in [13]. Here we will try to put these considerations on a firm ground, first showing explicitly the existence of a large number of flux compactifications in F-theory that include axions. These are afterwards translated to the type IIB compactifications on Calabi-Yau orientifolds, where the analysis of moduli stabilization is better understood. Then we will be able to generalize the existing stabilization techniques in order to accommodate for the new moduli.

For this reason we first try to give a brief introduction to type IIB flux compactifications in section 2, including the axions in the general discussion. In section 3 we discuss the stabilization procedure at tree level. We then show how stabilization changes after including perturbative and D3-instanton corrections, in sections 4 and 5 respectively. We comment on both the supersymmetric (KKLT) and non-supersymmetric (LVS) type of vacua. Based on [14–16] we are also able to estimate the importance of the worldsheet instantons on the moduli potential in section 6 and we see that the B_2 -moduli might substantially alter the moduli stabilization procedure in the large volume limit. We conclude by listing the possible applications of the axion moduli and suggestions for further research in section 7. Some of the more technical calculations used in the main text are carried out in the appendices.

2. Flux compactifications in type IIB string theory

We will first briefly review flux compactifications of type IIB string theory establishing the basic conventions and equations that will be used later.

The particle content of the type IIB supergravity is derived from the massless spectrum of the corresponding superstring type. The fermionic part consists of two left-handed Majorana-Weyl gravitinos and two right-handed Majorana-Weyl dilatinos. As supersymmetry holds and all fermionic degrees of freedom correspond exactly to bosonic ones, specifying either part of the effective action completely determines the other one. In this case there are 32 supersymmetry generators, i.e. we are in the case of $\mathcal{N} = 2$ supergravity in 10 dimensions. We will then concentrate on the bosonic part from here on, keeping in mind the fermionic counterparts. In the bosonic spectrum we have NS-NS and R-R bosons. The NS-NS bosons are the metric g_{MN} , a two-form B_2 (with corresponding field strength $H_3 = dB_2$) and the dilaton ϕ . The R-R sector consists of corresponding form fields C_0, C_2 , and C_4 , the latter having a self-dual field strength F_5 (also $F_1 = dC_0$ and $F_3 = dC_2$). In order to obtain four dimensional models with $\mathcal{N} = 1$ we need to compactify the theory on

Calabi-Yau orientifold where fluxes are turned on under the conditions:

$$\frac{1}{(2\pi)^2\alpha'} \int_{\Sigma_\alpha} F_3 = n_\alpha \in \mathbb{Z}, \quad \frac{1}{(2\pi)^2\alpha'} \int_{\Sigma_\beta} H_3 = m_\beta \in \mathbb{Z}, \quad (2.1)$$

with $\Sigma_{\alpha,\beta}$ three-cycles on the manifold. The resulting metric becomes a warped product of flat four-dimensional spacetime and conformally Calabi-Yau orientifold.

The compactification as described in this picture essentially requires a Calabi-Yau three-fold with $O3/O7$ orientifold planes, $D3/D7$ branes and the fluxes from (2.1). There is however another description of the same physical situation if one considers F-theory on an elliptically fibered Calabi-Yau four-fold [17]. There one needs to add only $D3$ branes and fluxes and the theory is equivalent to the one of type IIB flux compactification. Since in this way one obtains the orientifold "for free" without the need of explicitly constructing $O3/O7$ projection as in the type IIB picture, the F-theory approach is widely used for realistic constructions. The rules of translating between the two pictures are simple to use. A detailed summary can be found in section 4.1 of [18]. Here we will need to know that $(h^{1,1}(CY_4) - 1)$ corresponds to $h_+^{1,1}$ and $h^{2,1}(CY_4)$ to $h_-^{1,1}$, where $h_{+,-}^{1,1}$ are the Hodge numbers on the Calabi-Yau orientifold counting the even resp. odd parts of the $(1,1)$ -homology under the orientifold projection. The tadpole cancellation condition that needs to be satisfied in the F-theory picture is:

$$\frac{1}{(2\pi)^2\alpha'} \int H_3 \wedge F_3 + N_{D3} - N_{\bar{D}3} = \frac{\chi(CY_4)}{24}, \quad (2.2)$$

where χ is the Euler number of the four-fold. In the type IIB picture this number effectively collects the contribution to the $D3$ brane charge from the orientifold planes and the $D7$ branes. Clearly, $\chi(CY_4)$ needs to be divisible by 24, which puts a restriction on the space of elliptic four-folds that can be used for compactification (not too strict one since $\chi(CY_4) = 48 + 6(h^{1,1} + h^{3,1} - h^{2,1})$).

The resulting effective field theory corresponds to a standard $\mathcal{N} = 1$ supergravity with number of scalar (moduli) fields counted by the Hodge numbers. The KKLТ and LVS scenarios, as well as the vast literature on the subject of type IIB moduli stabilization, focus the attention on breaking the no-scale structure of the potential and on stabilizing the Kähler moduli at a value where the internal manifold has a large volume as consistency requires. In this process the non-geometric Kähler moduli are usually completely disregarded and assumed non-existent. This is only justified in special cases for orientifold projections where $h_-^{1,1} = 0$, as otherwise we have additional moduli coming from the 2-form fields B_2 and C_2 of the type IIB low energy effective action. One can find many examples of Calabi-Yau four-folds leading to both $h_-^{1,1} = 0$ and $h_-^{1,1} \neq 0$ (cf. [19] or table B.4 of [20] - keep in mind that $h_-^{1,1} = h^{2,1}(CY_4)$).

For a generic manifold ($h_-^{1,1} \neq 0$), the moduli to be stabilized in the theory are the axio-dilaton $\tau = C_0 + ie^{-\phi}$ (from here on referred to simply as dilaton), $h_-^{2,1}$ complex scalars z^i parametrizing the size of the surviving three-cycles appearing in (2.1), the Kähler moduli:

$$J = v^\alpha(x)\omega_\alpha(y), \quad \alpha = 1, \dots, h_+^{(1,1)}, \quad (2.3)$$

and the corresponding axionic moduli ρ_α from the four-form C_4 :

$$C_4 = \rho_\alpha(x)\tilde{\omega}^\alpha(y), \quad a = 1, \dots, h_+^{(1,1)}, \quad (2.4)$$

with $\{\tilde{\omega}^\alpha\}$ the basis of harmonic $(2, 2)$ -forms, dual to the $(1, 1)$ basis $\{\omega_\alpha\}$ that is even under the orientifold projection. The additional moduli entering the effective four-dimensional field theory because of $h_-^{1,1}$ are:

$$B_2 = b^a(x)\omega_a(y), \quad C_2 = c^a(x)\omega_a(y), \quad a = 1, \dots, h_-^{(1,1)}, \quad (2.5)$$

where $\{\omega_a\}$ is the basis of harmonic $(1, 1)$ forms that are odd under the orientifold projection. In the above formulae, x denotes the four-dimensional space-time where all the moduli (and we) live, and y are the coordinates on the compact six-dimensional internal manifold.

With these definitions, the Kähler metric on the space of moduli fields is given in terms of the reduced complex structure coordinates coming from the explicit manifold and in terms of the dilaton, the Kähler and the axionic moduli arranged as follows [21, 22]:

$$\begin{aligned} \tau &= C_0 + ie^{-\phi}, & G^a &= c^a - \tau b^a, \\ T_\alpha &= \frac{3i}{2}\rho_\alpha + \frac{3}{4}\kappa_\alpha(v) + \frac{3i}{4(\tau - \bar{\tau})}\kappa_{\alpha ab}G^a(G - \bar{G})^b, \end{aligned} \quad (2.6)$$

where $\kappa_\alpha(v) \equiv \kappa_{\alpha\beta\gamma}v^\beta v^\gamma$, i.e. it is just a four-cycle volume (with a different normalization compared to the standard literature, used for simplicity). In this notation,

$$\kappa \equiv \kappa_\alpha v^\alpha = 6V_{\text{CY}} = \kappa_{\alpha\beta\gamma}v^\alpha v^\beta v^\gamma, \quad (2.7)$$

where V_{CY} is the volume of the manifold already after the orientifold projection. The numbers $\kappa_{\alpha\beta\gamma}$ and $\kappa_{\alpha ab}$ are the usual Calabi-Yau intersection numbers after performing the orientifold projection. As explained in [22, 23], in the process of orientifolding consistency requires that only the intersection numbers with even number of Latin indices are non-zero. This means that for all α, β, a, b, c , $\kappa_{\alpha\beta a} = \kappa_{abc} = 0$ has to hold. The explicit construction of orientifolds with such properties might not be straightforward. However, we need not worry about this issue since orientifolding is performed implicitly from the F-theory picture and thus consistency is guaranteed.

The standard $\mathcal{N} = 1$ F-term potential¹ for the moduli fields is given by:

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \quad (2.8)$$

where the indices I, J run over all chiral fields (the ones defined through (2.6) together with the complex structure moduli z^i), the matrix $K^{I\bar{J}}$ is the inverse of the Kähler metric $K_{I\bar{J}} \equiv \partial_I \partial_{\bar{J}} K$, and $D_I W = \partial_I W + \partial_I K \cdot W$. Here, the Kähler potential K and the

¹Here we do not add D-terms that are also allowed in $\mathcal{N} = 1$ supergravity. These generally appear whenever there are charged chiral fields in the effective action. In principle this happens when one tries to reproduce the MSSM by adding D7-branes [24], but here we strictly concentrate on moduli stabilization and therefore neglect the possibility for a D-term potential.

Index	Chiral fields	Real scalars	Values
-	τ	C_0, ϕ	1
i	z^i	$Re(z^i), Im(z^i)$	$1, \dots, h_-^{2,1}$
α	T_α	$v^\alpha \leftrightarrow \kappa_\alpha, \rho_\alpha$	$1, \dots, h_+^{1,1}$
a	G^a	b^a, c^a	$1, \dots, h_-^{1,1}$

Table 1: Multiplicity of chiral and real moduli.

superpotential W are functions of the moduli fields in a particular way that will be discussed separately in the following sections. Once K and W are known, the moduli potential V can be calculated and the minima to which the moduli fields roll down and get stabilized can be found in principle.

From the above definitions, we see that:

$$\kappa_\alpha = \frac{2}{3}(T_\alpha + \bar{T}_\alpha) - \frac{i}{2(\tau - \bar{\tau})} \kappa_{\alpha ab} (G - \bar{G})^a (G - \bar{G})^b. \tag{2.9}$$

Had we assumed that $h_-^{1,1} = 0$ the additional G^a -dependent term would vanish and everything would be the same as in [3], so we see that the results in the literature are consistent with the neglect of the non-geometric moduli. However, if we really want to stabilize all moduli in the generic case where $h_+^{1,1} \sim h_-^{1,1} \sim \mathcal{O}(100)$ we need to use the coordinate basis given by (2.6). We will then describe in detail what happens in this case and show how all these moduli will be eventually stabilized in a manner similar to the KKL T and LVS procedures. In what follows we separately discuss the resulting moduli potential and its stabilization for the tree-level case, and for the cases with added perturbative α' -corrections to K and then D3-instantons to W . In the end we will be also able to draw conclusions on how the addition of worldsheet instanton corrections to the Kähler potential can influence the stabilization process.

Note that once we derive the moduli potential from the Kähler metric in the basis of chiral fields $\{\tau, T_\alpha, G^a\}$, we will be able to switch to the basis of real scalars $\{C_0, \phi, v^\alpha, \rho_\alpha, b^a, c^a\}$ using (2.6). It will turn out that minimization of the potential is easier in this new basis since the volume of the Calabi-Yau $V_{CY} = \frac{1}{6} \kappa_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma$ will depend only on the two-cycle moduli v^α and not on the other scalars. Of course, once having stabilized all scalars one can always switch back to the initial chiral fields where the metric on the moduli space takes a simpler form. For additional clarity we present table 1, listing the chiral and real fields that appear in this work, their multiplicity and associated indices.

3. Tree level

At tree level, in four dimensional $\mathcal{N} = 1$ supergravity, the Kähler potential is (see e.g. [5])

$$K = -\ln[i \int_{CY} \Omega(z) \wedge \bar{\Omega}(\bar{z})] - \ln(-i(\tau - \bar{\tau})) - 2 \ln(V_{CY}), \tag{3.1}$$

where the $V_{CY} = \frac{\kappa_\alpha v^\alpha}{6}$ has to be regarded as a function of the true Kähler coordinates (2.6). For κ_α we use (2.9), while v^α can only be written in terms of the chiral fields implicitly

by inverting the quadratic relation $\kappa_\alpha = \kappa_{\alpha\beta\gamma}v^\beta v^\gamma$. The superpotential at tree level is independent of the Kähler and axionic moduli and is given by the famous Gukov-Vafa-Witten [25] flux superpotential

$$W(z^i, \tau) = \int_{\text{CY}} \Omega(z^i) \wedge (F_3 - \tau H_3). \tag{3.2}$$

A detailed calculation of the Kähler potential and the superpotential was carried out in [5] and generalized to all orientifolds in [22], both quantities follow from the $\mathcal{N} = 2$ dimensional reduction of the low-energy effective action before orientifolding. The full moduli potential can be calculated from eq. (2.8). It is important here to stress that the potential at tree-level is positive semi-definite. This is not directly obvious from the expression, but is nevertheless true as it comes from the reduction of the $\mathcal{N} = 2$, where it is manifestly positive definite (c.f. appendix A.2 of [5]). This means that any full minimum of the potential will be at $V = 0$ and local minima (if any) could be only of de Sitter type (at $V > 0$).

With this information, we can now try to investigate the explicit form of the potential. The somewhat involved calculation of the Kähler metric and its inverse are carried out in appendix A.1 and the results are in exact accordance with those in [22, 26]. One of the main results is given by the simple expression

$$K^{A\bar{B}} K_A K_{\bar{B}} = 4, \tag{3.3}$$

where the indices A, B run over τ, T_α, G^a , and not over the complex structure moduli z^i . One can roughly break this sum into two contributions - a part in which the dilaton is involved plus a part coming only from the G^a 's and the T_α 's as given by (A.8). This will be helpful when we want to search for minima of the moduli potential.

The moduli potential (2.8) can now be calculated easily from (A.7) and (3.2), but its minima cannot be found analytically and depend on the specific model. The only class of controlled minima is realized when we stabilize the complex structure moduli and the dilaton to a supersymmetric minimum, $D_{z^i}W = D_\tau W = 0$ - the same procedure used in the KKLT and LVS. Imposing $D_{z^i}W = D_\tau W = 0$ results in stabilizing all z^i 's and τ to be some function of $\frac{\kappa_{\alpha ab} v^\alpha b^a b^b}{V_{\text{CY}}}$ as it appears in K_τ (A.5). When $\kappa_{\alpha ab} v^\alpha b^a b^b = 0$ this dependence vanishes and z^i, τ are stabilized to constants as in the original KKLT. The moduli potential after fixing $D_{z^i}W = D_\tau W = 0$ becomes

$$V = \frac{e^K e^{-2\phi} |W|^2}{4V_{\text{CY}}^2} (\kappa_{\alpha ab} v^\alpha b^a b^b)^2. \tag{3.4}$$

It is manifestly positive semi-definite once more. Clearly we can reach the global minimum $V = 0$ if $\kappa_{\alpha ab} v^\alpha b^a b^b = 0$. In the initial Calabi-Yau three-fold, $\kappa_{\hat{\alpha}\hat{\beta}\hat{\gamma}} v^{\hat{\alpha}}$ has a signature $(1, h^{1,1} - 1)$ [27] (here we use the convention $(+, -)$ for matrix signature). After the projection, $\kappa_{\alpha\beta\gamma} v^\alpha$ is with signature $(1, h_+^{1,1} - 1)$ and $\kappa_{\alpha ab} v^\alpha$ with $(0, h_-^{1,1})$. Then the only solution of $\kappa_{\alpha ab} v^\alpha b^a b^b = 0$ that is meaningful (i.e. we cannot have all $v^\alpha = 0$ as the Calabi-Yau manifold will vanish) is to set $b^a = 0$ for all a . This is the only generic possibility for a Minkowski vacuum in this case, depicted on figure 1.

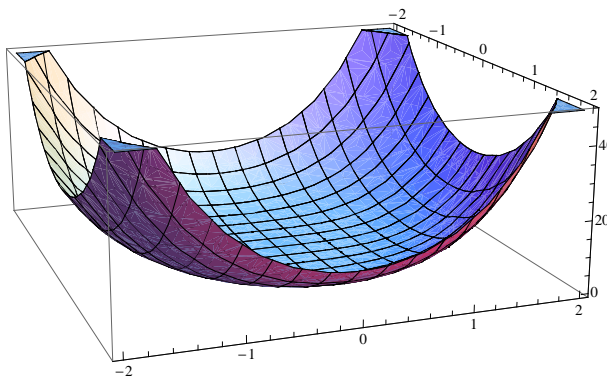


Figure 1: The form of the tree-level potential in the b^a -directions for $h_-^{1,1} = 2$.

Note that $V = 0$ can be also achieved for $V_{\text{CY}} \rightarrow \infty$, $e^{-\phi} = 0$, or $W = 0$. We are not interested in the first two cases as these contradict our initial construction, while $W = 0$ might be achieved for some solutions of $D_{z^i}W = D_\tau W = 0$ (in this case $\kappa_{\alpha ab}v^\alpha b^a b^b$ will be stabilized to a certain value since it appears in $D_\tau W$). $W = 0$ will correspond to a supersymmetric solution since then all covariant derivatives $D_I W$ vanish. However, it is not clear how often this is possible since the solutions of these equations cannot be given analytically, so the only generic solution remains $b^a = 0$ for all a .

Therefore we are very restricted in terms of possible analytic scenarios for stabilization of all moduli. The case when $\kappa_{\alpha ab}v^\alpha b^a b^b = 0$ is a generic minimum of the potential, corresponding to vanishing of all terms dependent on the non-geometric Kähler moduli. This mechanism leads us back to the no-scale potential that is flat in the directions of the geometric Kähler moduli since $V = 0$ after stabilizing all $b^a = 0$. Note that also the masses $m_{b^a} = 0$ in this case, which is not what we need as a final outcome.

So we need to improve our approach in order to break this no-scale behavior and lift up the axion mass. At this point one can employ the KKLT scenario of considering only D3-instantons and then stabilizing all moduli at a supersymmetric point. A special case of this idea was considered in [28]. We will however stick to the LVS procedure and calculate first the effect of the leading perturbative corrections and only afterwards of the instanton corrections on the potential that now includes the axionic moduli. This is in fact the more general case and it does not exclude, but only improves KKLT. Thus we will be able to consistently give mass to the b^a 's and c^a 's in the general case without the need to add D-terms in (2.8).

4. Perturbative α' -corrections

Including the leading perturbative α' -corrections as found first² in [8] by reducing to $\mathcal{N} = 1$

²Strictly speaking, only the orientifold with $h_-^{1,1} = 0$ was considered at first. Later it was shown in [22]

the results of [29] for the $\mathcal{N} = 2$ case, the Kähler potential becomes:

$$K = -\ln\left[i \int_{\text{CY}} \Omega(z) \wedge \bar{\Omega}(\bar{z})\right] - \ln(-i(\tau - \bar{\tau})) - 2 \ln \left(\frac{\kappa_\alpha v^\alpha}{6} + \frac{\xi}{2} \left(\frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \right), \quad (4.1)$$

where ξ is a constant, proportional to the Euler number of the CY three-fold:

$$\xi = -\frac{4\chi\zeta(3)}{(2\pi)^3}. \quad (4.2)$$

There are no α' -corrections to the superpotential in perturbation theory and so W is still given by (3.2). Even only the addition of corrections in K changes considerably the potential as we will see shortly. V does not have to be positive semi-definite any more since ξ could be either positive or negative depending on the sign of the Euler number of the Calabi-Yau. As we will see the sign of ξ will directly correspond to the sign of V .

To analyze the vacuum structure, we start again from (2.8). The computation of the inverse Kähler metric including the α' -corrections is given in appendix A.2 (see (A.16) and (A.17)). Thus once more we obtain a complicated expression³ for the potential that cannot be minimized in a controlled way. Similarly to the tree level case, we continue by imposing $D_{z^i}W = D_\tau W = 0$. At tree level, the stabilization of the other moduli then lead to minima at $V = 0$. This property does not hold any more when the α' -corrections are taken into account since the potential is no longer bounded from below. In the present case, we will know that we have found minima only if they are at large volumes (in string units) V_{CY} due to the argument given in the Large Volume Scenario [6]. It goes as follows. We write the full potential in a way to separate clearly the contributions from $D_{z^i}W$ and $D_\tau W$ from the other terms. So we split (2.8) in three terms - a quadratic with respect to $D_{z^i}W, D_\tau W$ (both summation indices in (2.8) run over z^i, τ), a linear (only one index including z^i or τ) and a constant (both indices running over the other moduli). Further we focus on the scaling of these terms with volume and thus we use the leading terms of the inverse Kähler metric (A.18):

$$V = e^K (K^{z^i \bar{z}^j} D_{z^i} W D_{\bar{z}^j} \bar{W} + K^{\tau \bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{W}) + \mathcal{O}(V_{\text{CY}}^{-2/3}) e^K (W D_{\bar{\tau}} \bar{W} + \bar{W} D_\tau W) + V_{\alpha'}, \quad (4.3)$$

with

$$V_{\alpha'} = \frac{e^K e^{-2\phi} |W|^2}{4V_{\text{CY}}} \left(3\xi e^{\phi/2} + \frac{(\kappa_{\alpha ab} v^\alpha b^a b^b)^2}{V_{\text{CY}}} + \mathcal{O}(V_{\text{CY}}^{-2/3}) \right). \quad (4.4)$$

In (4.4) we have given only the leading terms in large volume, because the complete analytic expression looks complicated (c.f. (A.17)) and we will only discuss large volume stabilization for the following reason. The first term of (4.3) is positive semi-definite and is only zero at the supersymmetric case $D_{z^i}W = D_\tau W = 0$. This term dominates the other two at large volumes as it scales as V_{CY}^{-2} while the two others scale as $V_{\text{CY}}^{-8/3}$ and V_{CY}^{-3}

that this can be trivially extended for a generic orientifold.

³Nevertheless, eq. (3.3) still holds. The factor 4 is generic for this class of Kähler potentials as discussed in [30].

respectively. Then any movement of the complex structure and dilaton moduli away from the supersymmetric point increases the potential, i.e. this point is a stable minimum. The moduli potential simply becomes $V = V_{\alpha'}$ and minimizing it with respect to v^α, b^a will result in full minimization of the initial moduli potential in all directions as long as the large volume assumption is satisfied for the obtained minima.

Therefore, we can consistently neglect the terms of order $V_{\text{CY}}^{-11/3}$ and lower in (4.4). We first observe that, apart from the non-generic supersymmetric point at $W = 0$ (corresponding to KKL_T type of extremum), we again need to set $\kappa_{\alpha ab} v^\alpha b^a b^b = 0 \Leftrightarrow \forall b^a = 0$ in order to minimize the term depending on the axionic moduli. But in this case we are still left with volume dependence since the ξ term survives. Now we see how important the sign of ξ turns out to be:

- $\xi > 0$, i.e. $\chi_{\text{CY}} < 0$: The resulting potential is positive definite and vanishing as $V_{\text{CY}} \rightarrow \infty$, i.e. this case is consistent with our assumptions but leads to decompactification of the Calabi-Yau. One can only hope that non-perturbative effects will eventually create a minimum at some finite large value of the volume (this is what happens in the LVS).
- $\xi < 0$, i.e. $\chi_{\text{CY}} > 0$: In this case the minimum is when the volume goes to zero and the potential goes to $-\infty$. Clearly, none of these is in accordance with the approximations made so far, and we can only trust the result at large volumes where no minima can be found. Instanton corrections cannot help in generating large volume minima since they cannot uplift the global minimum at $V_{\text{CY}} = 0$. Therefore, this case is undesirable and one needs very different approach in order to solve the problem of stabilizing the moduli for positive Euler number Calabi-Yau three-folds.

5. D-brane instanton corrections

Until now we only considered the tree-level superpotential (3.2). Let us see what happens if we assume that the compactification manifold meets the criteria that allow for nonzero D3-instanton contributions to W .

At this point a few words about instantons are in order. In string theory instantons can appear in Calabi-Yau compactifications when Euclideanized branes wrap cycles of the manifold [31]. If the branes wrap around cycles in such a way that supersymmetry is preserved, the corresponding cycle is called supersymmetric. It is exactly those cases that give a finite non-vanishing contribution to some of the physical quantities. As explained in [10], the counting of zero modes for a specific cycle eventually determines if it is supersymmetric or not. This translates into a nontrivial condition on the given cycle, depending on its dimension. For example (relevant here) it turns out that the 4-cycles that satisfy these criteria, admitting D3-brane instantons, are the ones that have an Euler number $\chi_E = 1$. However, this condition is more subtle after the addition of fluxes [11, 12] and then one has to check each cycle separately. Fundamental string worldsheets as well as NS5-branes can also give rise to instantons. It turns out that worldsheet instantons give

rise to non-perturbative α' corrections to the Kähler potential, while D3-branes and NS5-branes contribute to the superpotential. In this paper we shall neglect NS5 contributions since they are subleading at large volume as discussed in [32].

The superpotential with D3-instanton corrections is then:

$$W = W_{\text{tree}} + \sum_{\alpha} A_{\alpha}(z^i, \tau, G^a) e^{-a_{\alpha} T_{\alpha}} = W_0 + W_{\text{np}}, \quad (5.1)$$

where the sum over α only goes through the supersymmetric cycles. The coefficients A_{α} can in principle depend on all other moduli except the T_{α} 's but their explicit dependence is hard to determine and does not lead to further insight in the process of moduli stabilization at present (see, e.g. section 2.4 of [23]).

We can directly use the Kähler potential (4.1) since we already showed that the α' -corrections will substantially change the minimization process and cannot be neglected. Therefore, the moduli potential in analogy to (4.3) will become:

$$V = e^K (K^{z^{\alpha} \bar{z}^{\beta}} D_{z^{\alpha}} W D_{\bar{z}^{\beta}} \bar{W} + K^{\tau \bar{\tau}} D_{\tau} W D_{\bar{\tau}} \bar{W}) + \mathcal{O}(V_{\text{CY}}^{-2/3}) e^K (W D_{\bar{\tau}} \bar{W} + \bar{W} D_{\tau} W) + V_{\alpha'} + V_{\text{np}1} + V_{\text{np}2}, \quad (5.2)$$

with

$$V_{\alpha'} = \frac{e^K e^{-2\phi} |W|^2}{4V_{\text{CY}}} \left(3\xi e^{\phi/2} + \frac{(\kappa_{\alpha ab} v^{\alpha} b^a b^b)^2}{V_{\text{CY}}} + \mathcal{O}(V_{\text{CY}}^{-2/3}) \right), \quad (5.3)$$

$$V_{\text{np}1} = e^K \sum_{\alpha, \beta} \left\{ \left(-\frac{3}{2} (\kappa \kappa_{\alpha\beta} - \frac{3}{2} \kappa_{\alpha} \kappa_{\beta}) - \frac{3}{2} e^{-\phi} \kappa \kappa^{ab} \kappa_{\alpha ac} b^c \kappa_{\beta bd} b^d + \mathcal{O}(\kappa^0) \right) a_{\alpha} a_{\beta} A_{\alpha} \bar{A}_{\beta} + \right. \\ \left. + (-i e^{-\phi} \kappa \kappa^{ab} \kappa_{\alpha bc} b^c + \mathcal{O}(\kappa^0)) (a_{\alpha} A_{\alpha} \partial_{\bar{G}^a} \bar{A}_{\beta} - a_{\alpha} \bar{A}_{\alpha} \partial_{G^a} A_{\beta}) + \right. \\ \left. + \left(-\frac{2}{3} e^{-\phi} \kappa \kappa^{ab} + \mathcal{O}(\kappa^0) \right) \partial_{G^a} A_{\alpha} \partial_{\bar{G}^b} \bar{A}_{\beta} \right\} e^{-(a_{\alpha} T_{\alpha} + a_{\beta} \bar{T}_{\beta})}, \quad (5.4)$$

$$V_{\text{np}2} = e^K \sum_{\alpha} \left(\frac{3}{2} \kappa_{\alpha} + \mathcal{O}(\kappa^{-2/3}) \right) \left(a_{\alpha} A_{\alpha} \bar{W} e^{-a_{\alpha} T_{\alpha}} + a_{\alpha} \bar{A}_{\alpha} W e^{-a_{\alpha} \bar{T}_{\alpha}} \right), \quad (5.5)$$

where the summations are still only over supersymmetric cycles. $V_{\text{np}1} = e^K K^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W}$ and $V_{\text{np}2} = e^K K^{i\bar{j}} (K_i W \partial_{\bar{j}} \bar{W} + \partial_i W K_{\bar{j}} \bar{W})$, $i, j = T_{\alpha}, G^a$ are new terms here - they appear because now $\partial_{T_{\alpha}} W \neq 0$ and $\partial_{G^a} W \neq 0$.

By the same argument from the discussion after eq. (4.4), at large volumes we can consistently set $D_{z^{\alpha}} W = D_{\tau} W = 0$. The resulting equations have G^a and T_{α} dependence that is suppressed with V_{CY} , so we can safely assume that all complex structure moduli and the dilaton have been set to constants. Then,

$$V = V_{\alpha'} + V_{\text{np}1} + V_{\text{np}2}. \quad (5.6)$$

In principle, at this point we can also choose to follow the KKLT proposal and stabilize all moduli supersymmetrically, i.e. requiring additionally $D_{T_{\alpha}} W = D_{G^a} W = 0$. The solutions of these equations will correspond to a set of extrema of the potential and one has to check explicitly which ones are minima. Thus we would obtain a number of solutions

to our problem that unfortunately cannot be listed analytically and so we cannot draw any further conclusions. Therefore we now turn to the LVS idea of trying to minimize (5.6) at large manifold volume, which ensures us of finding minima in the full moduli space.

However, it is not so straightforward to minimize (5.6) and we need to make some simplifications of V_{np1} and V_{np2} in order to proceed. The scaling of both expressions (5.4) and (5.5) is being dominated by the exponential terms, and more precisely by the real part of the term in the exponent, while the imaginary part decides on the sign. At large 4-cycle volumes the terms are very suppressed and we can safely ignore them as the exponential function drops to zero very rapidly.⁴ Therefore the dominating terms in V_{np1} and V_{np2} will be the ones corresponding to the small (supersymmetric) cycles κ_α , which we shall denote κ_s . Here we implicitly assume that the internal manifold is of "Swiss-cheese" type [7], ensuring that small enough cycles do exist for large overall volume. This is required so that the new terms V_{np1} and V_{np2} can compete with the previously discussed $V_{\alpha'}$ as otherwise D-instanton corrections are diminishing and we arrive back at the situation of section 4.

5.1 One small 4-cycle

Assuming for the moment that there is one small 4-cycle and all the others are too big, in the sense that $e^{-\kappa_\alpha} \ll e^{-\kappa_s}$ for all $\alpha \neq s$, we finally obtain

$$V = e^K \left[-\alpha(b)\kappa_s e^{-\frac{3}{4}a_s\kappa_s} e^{\frac{3}{4}a_s e^{-\phi}\kappa_{sab}b^ab^b} + \frac{\beta(b)}{V_{CY}} + \gamma(b)(-\kappa_{ss})V_{CY} e^{-\frac{3}{2}a_s\kappa_s} e^{\frac{3}{2}a_s e^{-\phi}\kappa_{sab}b^ab^b} \right], \quad (5.7)$$

where the exact dependence of α , β and γ on the b^a 's is coming from (5.3) - (5.5):

$$\alpha(b) = -\frac{3}{2} \left(A_s \bar{W} e^{-ia_s(\frac{3}{2}\rho_s + \frac{3}{4}\kappa_{sab}(C_ob^a - c^a)b^b)} + \bar{A}_s W e^{ia_s(\frac{3}{2}\rho_s + \frac{3}{4}\kappa_{sab}(C_ob^a - c^a)b^b)} \right), \quad (5.8)$$

$$\beta(b) = \frac{e^{-2\phi}|W|^2}{4} \left(3\xi e^{\phi/2} + \frac{(\kappa_{\alpha ab}v^\alpha b^a b^b)^2}{V_{CY}} \right), \quad (5.9)$$

$$\gamma(b) = 6 \left[\left(\frac{3}{2} + \frac{3e^{-\phi}}{2\kappa_{ss}} \kappa^{ab} \kappa_{sac} b^c \kappa_{sbd} b^d \right) a_s^2 |A_s|^2 + \frac{2e^{-\phi}}{3\kappa_{ss}} \kappa^{ab} \partial_{G^a} A_s \partial_{\bar{G}^b} \bar{A}_s + \frac{ie^{-\phi}}{\kappa_{ss}} \kappa^{ab} \kappa_{sbc} b^c a_s (A_s \partial_{\bar{G}^a} \bar{A}_s - \bar{A}_s \partial_{G^a} A_s) \right]. \quad (5.10)$$

We further need to assume $\kappa_{ss} \simeq -\sqrt{\kappa_s}$ a la LVS, in order to make sure the γ term in (5.7) is not subleading. This is the only possibility to obtain large volume minima within the approximation of neglecting multi-instanton contributions to (5.1), as proven in details in the appendix of [33].⁵

In (5.8) for the first time we explicitly see some dependence on the moduli ρ_s, c^a defined through (2.4) and (2.5). This means we are allowed to stabilize them in a way that will maximize α , thus minimizing the overall potential. Since they appear only in the imaginary

⁴Strictly speaking, we are cheating here. Even for the large cycles κ_L , big enough values of $\kappa_{Lab}b^ab^b$ will make the non-perturbative contributions important. We will neglect such possibility at first and comment on it when we consider the general case with many 4-cycles in subsection 5.2.

⁵Multi-instanton contributions can be safely ignored as long as $a_s\kappa_s \gg 1$ in string units.

part of the exponent they can only determine the sign of α but not its magnitude (they can give a relative prefactor between -1 and 1). Therefore it is clear that ρ_s, c^a arrange themselves in a way to make the expression as large positive as possible. Since they appear in the term $a_s A_s \bar{W} e^{-ia_s(\frac{3}{2}\rho_s + \frac{3}{4}\kappa_{sab}(C_o b^a - c^a)b^b)} + c.c.$, there will be one equation to constrain the possible values of ρ_s and the c^a 's. This will be enough to stabilize ρ_s as in the original LVS and the c^a 's still remain unstabilized. Therefore $\alpha > 0$ with certainty and its b^a -dependence is absorbed by ρ_s , such that

$$\alpha = 3|A_s||W|.$$

On the other hand, we know that $\gamma(b)$ must be positive as it comes from the inner product of the vector $\partial_i W$ with itself. $\beta(b)$ is also positive by assumption since a negative value will not lead to consistent minima as shown in the previous section. Then, in order to minimize the full potential, the remaining free moduli will try to make the magnitude of the terms with β and γ as small as possible and the magnitude of the term with α as big as possible (as it appears with negative sign).

To find the minima of the potential V we need to solve the system of equations $\frac{\partial V}{\partial b^a} = 0$ for all a , $\frac{\partial V}{\partial \kappa_s} = 0$, and $\frac{\partial V}{\partial V_{CY}} = 0$. To leading orders in volume,

$$\begin{aligned} \frac{\partial V}{\partial b^a} = & e^K \kappa_{sab} b^b e^{-\frac{3}{4}a_s \kappa_s} e^{\frac{3}{4}a_s} e^{-\phi \kappa_{sab} b^a b^b} \sqrt{\kappa_s} \left[-\frac{3}{2} \alpha \sqrt{\kappa_s} + 3\gamma V_{CY} e^{-\frac{3}{4}a_s \kappa_s} e^{\frac{3}{4}a_s} e^{-\phi \kappa_{sab} b^a b^b} \right] + \\ & + \frac{e^K e^{-2\phi} |W|^2}{V_{CY}^2} \kappa_{\alpha ab} v^\alpha b^b \kappa_{\alpha ab} v^\alpha b^a b^b + e^K \frac{\partial \gamma}{\partial b^a} V_{CY} e^{-\frac{3}{2}a_s \kappa_s} e^{\frac{3}{2}a_s} e^{-\phi \kappa_{sab} b^a b^b}. \end{aligned} \quad (5.11)$$

We have extrema of the potential in the b -moduli directions whenever $\frac{\partial V}{\partial b^a} = 0$ for all b^a . This is satisfied by $b^a = 0$ for all a ,⁶ while other solutions can be found only for specific cases depending on the form of the intersection numbers $\kappa_{\alpha ab}$ and the coefficients A_s .

Note that if all $b^a = 0$ we get back the Large Volume Scenario [6], $\alpha(b) = \alpha_{LVS}$, $\beta(b) = \beta_{LVS}$, and $\gamma(b) = \gamma_{LVS}$. The solutions of $\frac{\partial V}{\partial \kappa_s} = 0$ and $\frac{\partial V}{\partial V_{CY}} = 0$ can be found explicitly only numerically, but the small cycle will be always stabilized to $\kappa_s \approx \ln(V_{CY})$. Then minima at large volume V_{CY} can exist under the same conditions as in [6, 7], i.e. some particular relative weight of the prefactors α , β , and γ ($\beta \gg \alpha$ and/or $\gamma \gg \alpha$).⁷

If all $b^a = 0$ we can go further and compute the matrix of second derivatives:

$$\left(\frac{\partial^2 V}{\partial b^a \partial b^b} \right)_{b^a=0, \forall a} = \frac{3e^K}{V_{CY}} \left[\kappa_{sab} \left(-\frac{1}{2} \alpha \ln(V_{CY}) + \gamma \sqrt{\ln(V_{CY})} \right) - \frac{e^{-\phi}}{2} \kappa^{cd} \kappa_{sac} \kappa_{sbd} \right]. \quad (5.12)$$

⁶Here we further assume that $\partial_{G^a} A_s = 0$ when all $b^a = 0$. Thus, $\frac{\partial \gamma}{\partial b^a} = 0$ at this point of moduli space. If this is not the case, $b^a = 0, \forall a$ cannot be an extremum of V , but $\frac{\partial \gamma}{\partial b^a}$ will still be small at large volumes and the extremum will be very close to $b^a = 0, \forall a$ without changing our qualitative discussion.

⁷It is easy to see from (5.7) that the term with α will always dominate at $V_{CY} \rightarrow \infty$ as it will scale additionally as $\ln(V_{CY})$, while the term with γ only scales with $\sqrt{\ln(V_{CY})}$ and the term with β has no additional scaling. Thus large volume minima can only be reached when one of the positive β and γ terms competes and dominates over the negative term until V_{CY} is large. So we can roughly estimate the relative weights of α, β, γ based on scaling. We need $V_{CY} \gtrsim 10^6$, thus $\beta \gtrsim 14\alpha$ and/or $\gamma \gtrsim 4\alpha$. In the main text we denote these criteria $\beta \gg \alpha$ and/or $\gamma \gg \alpha$ in order to keep the discussion as general as possible.

$\kappa^{cd}\kappa_{sac}\kappa_{sbd}$ is a negative definite matrix since $\kappa^{cd} = (\kappa_{cd})^{-1}$ (here we use the definition $\kappa_{ab} \equiv \kappa_{\alpha ab}v^\alpha$), so it always gives a positive contribution to (5.12). However, this term is subleading in V_{CY} and therefore we concentrate on the other part of the expression. In typical cases α and γ are of the same order of magnitude so the term in round brackets is negative.⁸ The matrix κ_{sab} could in some cases be negative definite as it comes from the orientifold projection and we know from before that $\kappa_{\alpha ab}v^\alpha$ is negative definite. If this is the case, $\left(\frac{\partial^2 V}{\partial b^a \partial b^b}\right)_{b^a=0, \forall a}$ is positive definite and therefore $b^a = 0$ is a minimum of the potential with the Large Volume Scenario holding for suitable values of α, β, γ . In principle, even when κ_{sab} has nonnegative eigenvalues we can have full minima due to the positive contribution from $\kappa^{cd}\kappa_{sac}\kappa_{sbd}$ but this seems possible only for not so large values of V_{CY} and is therefore not a generic case. The argument is reversed when $\gamma \gg \alpha$, as in this case the term in the round brackets becomes positive and $b^a = 0$ is a minimum if κ_{sab} is positive definite (not very likely).

Apart from this analytic class of minima, we can show the existence of another class of minima, for which explicit solutions cannot be given. From (5.11) we see there can be extrema also when $b^a \neq 0$ for some a 's. And, in fact, we know that some of these extrema will certainly be minima of the potential as long as $b^a = 0$ is not a minimum. The proof that there is always at least one minimum of the potential is carried out in appendix B. It follows that if κ_{sab} does not satisfy the above conditions to make $b^a = 0$ a minimum, then there will still be a minimum with at least one of the b^a 's nonzero. In this case we lose analytic control over the values of κ_s and V_{CY} at the minimum, so we cannot a priori make sure that the large volume assumption and the neglect of multi-instanton contributions are justified. This will fully depend on the explicit form of the intersection numbers $\kappa_{\alpha ab}$. It is only clear that the b^a 's will still tend to minimize $\kappa_{\alpha ab}v^\alpha b^a b^b$ in (5.9), i.e. as many as possible of the b -moduli will be zero if they are not fixed by $\frac{\partial V}{\partial b^a} = 0$. One would naturally expect that the closer $\kappa_{sab}b^a b^b$ is to zero at the minimum, the closer the values of κ_s and V_{CY} are to the LVS case.

To illustrate the above explicitly, consider a simple version of (5.11) where $\frac{\partial \beta}{\partial b^a}, \frac{\partial \gamma}{\partial b^a}$ always vanish. Then another analytic solution of $\frac{\partial V}{\partial b^a} = 0$ is

$$\kappa_{sab}b^a b^b = \frac{4e^\phi}{3a_s} \ln \left(\frac{\alpha\sqrt{\kappa_s}}{2\gamma V_{CY}} \right) + e^\phi \kappa_s.$$

This is satisfied generally on a hypersurface of the full b -moduli space where at least one of the b^a 's is nonzero. Lower order corrections will then also fix the remaining free b^a 's. It is easy to verify that this hypersurface is a minimum in all b -directions, but when considering minimization in the κ_s and V_{CY} directions this is no longer a valid solution, as expected since our initial assumption to neglect the b -dependence of β and γ is clearly wrong. However, this gives us some intuition for what to expect roughly from the possible minima that are not at $b^a = 0, \forall a$. It is likely that brute-force solution of (5.11) will only lead to a hypersurface of minima that is subsequently refined by the lower order corrections.

⁸ α and γ are determined from the stabilization of z^i and τ , so "typical" here refers to statistically more probable. $\gamma \gg \alpha$ only when W_0 is small.

Generic minimum	$\alpha \leftrightarrow \gamma$	Restr. on κ_{sab}	Large Volume
$b^a = 0, \quad \forall a$	$\alpha\sqrt{\ln V_{\text{CY}}} > 2\gamma$ $\alpha\sqrt{\ln V_{\text{CY}}} < 2\gamma$	Neg. def. Pos. def.	$\beta \gg \alpha$ $\beta \gg \alpha / \gamma \gg \alpha$
$\frac{\partial V}{\partial b^a} = 0, \quad a = 1, \dots, h_-^{1,1}$ $\exists \tilde{a}, \text{ s.t. } b^{\tilde{a}} \neq 0$	$\alpha\sqrt{\ln V_{\text{CY}}} > 2\gamma$ $\alpha\sqrt{\ln V_{\text{CY}}} < 2\gamma$	Not neg. def. Not pos. def.	$\beta \gg \alpha$ $\beta \gg \alpha / \gamma \gg \alpha$

Table 2: Axion stabilization scenarios for one small 4-cycle s .

So finally we emerge with two main scenarios for stabilization of the non-geometric moduli that entirely depend on the specific Calabi-Yau intersection numbers. The two cases are sketchily summarized in table 2. If κ_{sab} has also zero eigenvalues there will be flat directions at leading order, which will be fixed by the subleading tree level term $\kappa_{\alpha ab} v^\alpha b^a b^b$ in β (we will see soon that for more small moduli flat directions at leading order are unlikely to appear).

5.2 Many small 4-cycles

Generalizing these conclusions for many small moduli is more involved due to a subtlety coming from V_{np1} (see (5.4)). There we obtain a mix of exponential terms for different cycles. Now also each separate small four-cycle (as long as it is supersymmetric) will lead to a corresponding non-perturbative contribution to the α and γ terms:

$$\begin{aligned}
 V = e^K & \left[\frac{\beta(b)}{V_{\text{CY}}} + \sum_{i=1}^n \left\{ -\alpha_i(b) \kappa_{s_i} e^{-\frac{3}{4} a_{s_i} \kappa_{s_i}} e^{\frac{3}{4} a_{s_i} e^{-\phi} \kappa_{s_i ab} b^a b^b} \right. \right. \\
 & \left. \left. + \gamma_i(b) (-\kappa_{s_i s_i}) V_{\text{CY}} e^{-\frac{3}{2} a_{s_i} \kappa_{s_i}} e^{\frac{3}{2} a_{s_i} e^{-\phi} \kappa_{s_i ab} b^a b^b} \right\} \right] \quad (5.13) \\
 & - \sum_{i < j} \left\{ 6 V_{\text{CY}} e^K \left[i e^{-\phi} \kappa^{ab} \kappa_{s_i bc} b^c (a_{s_i} A_{s_i} \partial_{\bar{G}^a} \bar{A}_{s_j} - a_{s_i} \bar{A}_{s_i} \partial_{G^a} A_{s_j}) + \frac{2}{3} e^{-\phi} \kappa^{ab} \partial_{G^a} A_{s_i} \partial_{\bar{G}^b} \bar{A}_{s_j} \right. \right. \\
 & \left. \left. + \left(\frac{3}{2} \kappa_{s_i s_j} + \frac{3}{2} e^{-\phi} \kappa^{ab} \kappa_{s_i ac} b^c \kappa_{s_j bd} b^d \right) a_{s_i} a_{s_j} A_{s_i} \bar{A}_{s_j} \right] e^{-(a_{s_i} T_{s_i} + a_{s_j} \bar{T}_{s_j})} + c.c. \right\},
 \end{aligned}$$

where the constants α_i and γ_i are defined in analogy to (5.8), (5.10) with addition of the index i where needed to distinguish between different small-volume cycles. Thus we can stabilize all ρ_{s_i} by maximizing each α_i separately. We see that the second part of (5.13) (third and fourth row) is a new term that mixes in a complicated way all moduli $\kappa_{s_i}, \rho_{s_i}, b^a, c^a$ (hidden in the exponents of T_{s_i}). Its value is ultimately restricted by the condition $V_{np1} \geq 0$ so it must be smaller than the γ_i contributions. These additional terms will solve the problem with the unstabilized c^a 's as they exhibit a nontrivial dependence on them (unless all $b^a = 0$). Clearly, the minima with respect to the c^a 's can only be found numerically after specifying the concrete model and the number of stabilized axions will depend on the values of $h_-^{1,1}$ and the cycles admitting instanton corrections. Therefore the stabilization of c^a 's cannot be controlled analytically very well, analogously to the stabilization of ρ_α 's.

As before, it is easy to see⁹ that there is an extremal point at $b^a = 0$ for all a : $(\frac{\partial V}{\partial b^a})_{b^a=0, \forall a} = 0$. Again, other analytic solutions of $\frac{\partial V}{\partial b^a} = 0$ cannot be given, here the equation is even more complicated than (5.11). When all $b^a = 0$, we recover the many-cycle LVS. The dependence on the c^a moduli of the potential disappears while dependence on ρ_{s_i} becomes considerably more complicated compared to the one small cycle case. A detailed discussion of the minimization in the ρ -directions is given in A.2 of [33] and we will not repeat it here. The main result in the end is that large volume minima as before can still exist for certain configurations of the intersection numbers (see the reference for more details): again, $\kappa_{s_i} \approx \ln(V_{CY})$, $(-\kappa_{s_i s_i}) \approx \sqrt{\ln(V_{CY})}$ for all small cycles and the volume is stabilized at a large value. This point is a minimum in the b -moduli directions as long as

$$\left(\frac{\partial^2 V}{\partial b^a \partial b^b}\right)_{b^a=0, \forall a} = \frac{3e^K}{V_{CY}} \left[\sum_{i=1}^n \kappa_{s_i ab} \left(-\frac{1}{2} \alpha_i \ln(V_{CY}) + \gamma_i \sqrt{\ln(V_{CY})} \right) + \mathcal{O}(V_{CY}^{-1/3}) \right] \quad (5.14)$$

is positive definite. Note that in general $\alpha_1 \simeq \alpha_2 \dots \simeq \alpha_n$ and $\gamma_1 \simeq \gamma_2 \dots \simeq \gamma_n$ as they differ only by the small differences in the proportionality constants a_{s_i}, A_{s_i} . Now the condition for $b^a = 0$ to be minimum essentially states that the combined matrix as sum of sub-matrices $(\kappa_{s_1 ab} + \dots + \kappa_{s_n ab})$ ¹⁰ should be negative or positive definite depending on the term in brackets (most likely $\alpha_i \sim \gamma_i$ and then the matrix needs to be negative definite).

Once again, we can in general prove the existence of at least one minimum. The argument goes exactly as in the case of one small modulus in appendix B, and the essential point will again be that the dependence on γ_i is not crucial asymptotically so we can neglect it (this also means neglect of the additional mixing terms as they arise together with the γ_i terms from (2.8)). We will not repeat the same considerations specifically for this case, as all statements in appendix B can be easily generalized to include many small moduli.

There is now an important difference between the analytical minimum $b^a = 0$ and the other possibility when at least one of the b^a 's is nonzero. In the latter case the additional mixing term will depend on the c^a 's and we will be able to fix some or all of them while also being able to stabilize the ρ_{s_i} 's in a more straightforward manner. The number of stabilized c^a 's will depend on the number of nonzero b -fields so one needs to go to the specific manifold model. Furthermore, when some b^a 's are nonzero there is an additional subtlety. The full potential originally depends also on the large cycle κ_L but we regarded this contribution as largely suppressed. However, if $\kappa_{Lab} b^a b^b \gg 0$ this assumption might not be correct and there would be another term to consider. If this is the case we can drop the requirement for "Swiss-cheese" manifold since we will no longer make use of a clear distinction between small and large 4-cycles. Again, this issue can only be assessed properly once an explicit manifold is chosen.

Note that in order to obtain a large volume minimum, the $\gamma_i \gg \alpha_i$ option is questionable in general due to the fact that the additional terms in (5.13) could decrease substan-

⁹Again, we assume that $\partial_{G^a} A_{s_i} = 0$ for all i when all $b^a = 0$.

¹⁰Here we assumed exact equalities $\alpha_1 = \alpha_2 \dots = \alpha_n$ and $\gamma_1 = \gamma_2 \dots = \gamma_n$. Generally, the combined matrix of interest is a weighted sum of $\kappa_{s_1 ab}, \dots, \kappa_{s_n ab}$, but the weights are nearly equal.

Generic minimum	Restr. on $\sum_i \kappa_{s_i ab}$	Large Volume	Stab. moduli
$b^a = 0, \quad \forall a$	Neg. def.	$\beta \gg \alpha_i, \forall i$	$z^i, \tau, V_{\text{CY}}, v^{s_i}, \rho_{s_i}$
$\frac{\partial V}{\partial b^a} = 0, \quad a = 1, \dots, h_-^{1,1}$ $\exists \tilde{a}, \text{ s.t. } b^{\tilde{a}} \neq 0$	Not neg. def.	$\beta \gg \alpha_i, \forall i$	$z^i, \tau, V_{\text{CY}}, v^{s_i}, \rho_{s_i},$ a part of $\{c^a\}$

Table 3: Axion stabilization scenarios for n small 4-cycles $s_1 \dots s_n$.

tially the effective value of each γ_i . Therefore, in order to make sure V_{CY} is stabilized large in all cases, we require that $\beta \gg \alpha_i, \forall i$.¹¹

We have thus found possible minima not only for vanishing b^a 's, but also for non-zero values. The main results are given in table 3. As we will see in the next section these minima could be further destabilized by other instanton effects, so the minimization of the axionic moduli turns out to be a nontrivial step in the stabilization process.

6. Worldsheet instanton corrections

Another correction to the Kähler potential is given by worldsheet instantons wrapping holomorphic 2-cycles on the internal manifold. It is inherited from the type IIA $\mathcal{N} = 2$ prepotential [14, 29], given by:¹²

$$F_0(X) = F_{\text{cl}}(X) + F_{\text{pert}}(X) + F_{\text{non-pert}}(X), \tag{6.1}$$

with

$$F_{\text{cl}}(X) = \frac{1}{3!} \frac{\kappa_{\alpha\beta\gamma} X^\alpha X^\beta X^\gamma}{X^0}, \quad F_{\text{pert}} = i \frac{\xi}{8} (X^0)^2,$$

$$F_{\text{non-pert}}(X) = i \frac{(X^0)^2}{(2\pi)^3} \sum_{\Sigma_\beta \in H_2} \sum_{n=1}^{\infty} \frac{n_{\Sigma_\beta}^0}{n^3} e^{2\pi n i k_\alpha^\beta X^\alpha / X^0}, \tag{6.2}$$

where ξ is as defined in (4.2), $\kappa_{\alpha\beta\gamma}$ are the usual intersection numbers, $k_\alpha^\beta = \int_{\Sigma_\beta} \omega_{\hat{\alpha}}$, $2X^\alpha \equiv i v^{\hat{\alpha}} + b^{\hat{\alpha}}, \alpha = 1, \dots, h_-^{1,1}$ before orientifolding,¹³ and X^0 has to be set to 1 after obtaining the Kähler potential. The numbers $n_{\Sigma_\beta}^0$ are the genus zero topological invariants of Gopakumar-Vafa [16], associated with each element of the homology. Thus, F_0 is the prepotential for the vector multiplets of type IIA at tree-level of string-loop expansion that

¹¹As in the one small 4-cycle case, for the non-analytic minima with nonzero b^a 's we cannot decide with certainty about the criterion for large volume minimum. We can only hope $\sum_i \kappa_{s_i ab} b^a b^b$ is not too far from zero and then use the same requirement $\beta \gg \alpha_i$ for all i .

¹²Note the slight change of notation as compared to [14, 29]. This is consistent with the intended identification of the coordinates X^I here and leads to the same form of the Kähler potential.

¹³The hats on $\hat{\alpha}$ are introduced for a clear distinction between the $\mathcal{N} = 1$ variables of section 2 and the ones used here at $\mathcal{N} = 2$.

receives both perturbative and non-perturbative corrections in α' .¹⁴ It can be translated to the type IIB orientifold picture by the classical c-map [34] with the new coordinates $X^\alpha = i v^\alpha/2, \alpha = 1, \dots, h_+^{1,1}$ and $X^a = b^a/2, a = 1, \dots, h_-^{1,1}$ (see also e.g. [15] for more details on how this works). The relevant part of the Kähler potential can then be calculated directly by

$$K = -2 \ln \left(i e^{-2\phi} \left(X^I \frac{\partial \bar{F}_0}{\partial \bar{X}^I} - \bar{X}^I \frac{\partial F_0}{\partial X^I} \right) \right)_{X^0=1}, \quad (6.3)$$

where the final result for K needs to be expressed as before in terms of the chiral fields of section 2. Using this, we finally obtain in the Einstein frame

$$K = -\ln(-i(\tau - \bar{\tau})) - 2 \ln \left(V_{\text{CY}} + \left(\frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \left(\frac{\xi}{2} + \frac{4}{(2\pi)^3} \varpi_{\text{ws}}(\tau, G) \right) \right), \quad (6.4)$$

with

$$\varpi_{\text{ws}}(\tau, G) = \sum_{\Sigma_\beta \in H_2^-} \sum_{n=1}^{\infty} \frac{n_{\Sigma_\beta}^0}{n^3} \cos \left(n\pi \frac{k_a^\beta (G - \bar{G})^a}{\tau - \bar{\tau}} \right) = \sum_{\Sigma_\beta \in H_2^-} \sum_{n=1}^{\infty} \frac{n_{\Sigma_\beta}^0}{n^3} \cos(n\pi k_a^\beta b^a), \quad (6.5)$$

where $k_a^\beta = \int_{\Sigma_\beta} \omega_a$ and ω_a are the harmonic $(1, 1)$ -forms that are odd under the orientifold projection and Σ_β are the corresponding 2-cycles. The contributions from the even $(1, 1)$ -forms are exponentially suppressed with the v^α 's and we can safely neglect them. This new Kähler potential includes infinite (converging) sum over n and another sum over the elements of the homology H_2^- of the CY manifold.¹⁵ This makes the metric very hard to invert and we cannot present a generic inverse of $K_{A\bar{B}}$ that is manifold independent as was the case before. However, we can use the intuition from previous results to draw quite generic conclusion on how worldsheet instantons can influence the moduli stabilization. Note that the corrections are subleading in volume,

$$K = -2 \ln(V_{\text{CY}}) - 2 \frac{\left(\frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \left(\frac{\xi}{2} + \frac{4}{(2\pi)^3} \varpi_{\text{ws}}(\tau, G) \right)}{V_{\text{CY}}} + O(V_{\text{CY}}^{-2}). \quad (6.6)$$

Therefore the worldsheet instantons will appear in the end result the same way as the perturbative corrections, i.e. in the definition of the β term (see eqs. (5.7) and (5.9)). They would be too subleading to influence the α, γ terms in (5.7).

If we consider more closely the dependence of ϖ_{ws} , we see that its extrema can be given by the condition $k_a^\beta b^a = l^\beta$ for an integer number l^β . In fact ϖ_{ws} is maximized for $l^\beta = 0, \pm 2, \pm 4 \dots$ and minimized whenever $l^\beta = \pm 1, \pm 3, \pm 5 \dots$ for every cycle Σ_β . If indeed $k_a^\beta b^a = l^\beta$ for all Σ_β , then both $\frac{\partial \varpi_{\text{ws}}}{\partial \tau}$ and $\frac{\partial \varpi_{\text{ws}}}{\partial C^a}$ vanish. So in this case we can effectively consider ϖ_{ws} to be constant for the purpose of obtaining analogs of (A.18) and (A.19) that

¹⁴These non-perturbative corrections are in fact the genus zero worldsheet instanton contributions to the prepotential. Higher genus worldsheets instantons do not appear in the prepotential and will not be discussed further. Although it is not fully precise, here we refer to the genus zero worldsheet instantons simply as worldsheet instantons.

¹⁵This sum also needs to be finite, see section 2.2 of [23] for discussion of this issue.

eventually determine the expression for β . Then the Kähler metric can be again inverted analytically¹⁶ just by adding the constant $\frac{8}{(2\pi)^3}\varpi_{\text{ws}}$ to the existing ξ in the formulae in appendix A.2. For the minimum of ϖ_{ws} we get:

$$\varpi_{\text{ws}}|_{k_a^\beta b^a=l^\beta, l^\beta=\pm 1, \pm 3, \dots, \forall \Sigma_\beta} = \sum_{\Sigma_\beta \in H_2^-} \sum_{n=1}^{\infty} \frac{(-1)^n n_{\Sigma_\beta}^0}{n^3} = -\frac{3\zeta(3)}{4} \sum_{\Sigma_\beta \in H_2^-} n_{\Sigma_\beta}^0. \quad (6.7)$$

The minimization of ϖ_{ws} means minimization of the full potential as it decreases the value of the β term:

$$\beta_{\text{ws}}|_{k_a^\beta b^a=l^\beta, l^\beta=\pm 1, \pm 3, \dots, \forall \Sigma_\beta} = \frac{3e^K e^{-3\phi/2} |W|^2 \zeta(3)}{2(2\pi)^3} \left((-2\chi) - 3 \sum_{\Sigma_\beta \in H_2^-} n_{\Sigma_\beta}^0 \right). \quad (6.8)$$

Note that in the case when $b^a = 0$ for all a , ϖ_{ws} is maximized and the sign in front of the instanton sum in (6.8) flips. This will make $b^a = 0$ much less likely to be a minimum of the potential, e.g. (5.12) and (5.14) will be corrected with the negative sum over Gopakumar-Vafa (GV) invariants. In case it is large enough, the sum will make sure that $b^a = 0, \forall a$ is in fact a maximum. And the minimum will certainly be at a point which decreases β . If the β term decreases so much that it becomes negative we will no longer have any consistent minima in the volume direction as discussed in sections 4 and 5. $\beta > 0$ is absolutely crucial for the existence of LVS minima, while supersymmetric KKLT minima can exist independently of the sign of β . On the other hand, if β is positive of the order of α, γ we will only have small volume minima in the LVS. So one can only hope that the 2-cycles of the manifold do not allow for larger values of the GV invariants and thus of the worldsheet instanton corrections as this can spoil the whole process of moduli stabilization. The case $\gamma \gg \alpha$ might still enable the existence of desired minima for small positive β , but this does not seem to be possible for many supersymmetric cycles as seen in subsection 5.2.

The above discussion is in fact quite general and does not necessarily have to hold only for the special points in b -moduli space that minimize ϖ_{ws} , although these are the cases that can be handled analytically (as long as the inverse Kähler metric is concerned). Even for generic values of the b^a 's at the minimum of the potential where we also get corrections from $\frac{\partial \varpi_{\text{ws}}}{\partial G^a}$, the β term will tend to decrease as all terms in β coming from worldsheet corrections will necessarily be periodic and therefore allowed to become negative.¹⁷ Note that in fact the worldsheet instantons are the leading term that exhibits b^a -dependence. The tree-level term from section 3 is suppressed by $V_{\text{CY}}^{1/3}$ compared to ϖ_{ws} , and therefore we expect that (even without having any D3-instantons) the b^a 's are stabilized away from zero, unless the GV invariants are small. Thus the volume will be usually stabilized at a lower value as compared to section 5, due to the decrease in the β term. As we see the risk for

¹⁶In the sense that the potential V is precise upto order $\mathcal{O}(V_{\text{CY}}^{-10/3})$, i.e. the inverse metric is analytic at leading order. This is all we need since we are working under the assumption of large volume. The Kähler metric cannot be inverted to all orders due to the fact that e.g. $\frac{\partial^2 \varpi_{\text{ws}}}{\partial G^a \partial G^b}$ does not vanish at the minimum.

¹⁷Unfortunately, as long as $|2\chi| < |3 \sum_{\Sigma_\beta} n_{\Sigma_\beta}^0|$, β will necessarily be stabilized negative as this ensures the minimum of the potential is at very large negative values $V_{\text{min}} \rightarrow -\infty$ and very small manifold volumes. This is of course not an acceptable vacuum as it contradicts all assumptions of our construction.

the stability of the LVS minima after adding worldsheet instantons is very general and one needs to explicitly calculate the invariants $n_{\Sigma_\beta}^0$ in order to make sure phenomenologically accepted vacua are still present in a given model. Even if this is so, the second type of generic vacuum in table 3 is the most plausible (and least well controlled) outcome.

7. Discussion

We made some progress towards full stabilization of the scalar fields in the compactification of type IIB string theory on Calabi-Yau orientifolds with $h_{-1,1} \neq 0$. As seen, the search for supersymmetric and non-supersymmetric minima of the moduli potential is a nontrivial task. Many approximations and simplifications are employed in the process and it is not always granted that these are justified in all possible models. Clearly, perturbative and non-perturbative corrections play an important role and it is unfortunate that at present there is no full classification of possible terms that can appear in the Kähler potential and the superpotential.

Nevertheless, in the literature one can find extensive discussion of quantum corrections and their regime of importance, i.e. how suppressed they are with the volume. References [9, 35], and [33] study this topic in detail and show that string loop corrections for "Swiss-cheese" CY manifolds are subleading compared to the perturbative and non-perturbative α' -corrections so they only help stabilizing the non-supersymmetric 4-cycle volumes, but there may be other types of manifolds for which this is not satisfied. Other possible perturbative α' -corrections are known to be less important compared to the ones discussed in the LVS, i.e. it seems that the LVS is safe from further perturbative α' and g_s corrections. However, there might be other corrections from DBI actions and $\mathcal{N} = 1$ supergravity that are of importance (see e.g. section 6 and 7 of [9] for more details). If we are to claim that realistic string compactifications have been found, a better understanding of all quantum corrections is needed. Needless to say, same holds for instanton corrections to W and K — as seen in the previous section worldsheet instantons have the potential to break down the LVS.

Despite these shortcomings, we managed to show with certainty that all b^a 's are stabilized already at tree level with instanton corrections possibly changing their vevs and lifting their masses, and that the ρ_α 's and (some of) the c^a 's are stabilized if D3-instanton effects contribute to the moduli potential of the given model. The above is true under the condition that the manifold volume is stabilized large, which ultimately depends on the topological data for the manifold and the stabilization of the complex structure moduli that appear implicitly in the α , β , and γ terms defined through (5.7)–(5.10). Therefore our procedure works for a subset of all minima that one can find in the landscape of vacua, i.e. for those cases that produce the desired relative weights of α, β, γ as discussed in section 5. How large this subset is depends on the specific Calabi-Yau manifold, which also determines the type of perturbative and non-perturbative corrections that should be considered. So in the end everything can be determined from the topological structure of the compactification manifold as expected. It seems that at present the full generality of the

construction defined in section 2 is exhausted and one needs to go to specific examples in order to obtain more explicit results that can be used for predictive purposes.

We will now try to briefly describe some applications that make use of these axionic moduli [36–38]. In type IIB the axions arise from the 2-form fields B_2 and C_2 and the 4-form field C_4 as given by eqs. (2.5) and (2.4). There are a few ideas to employ these scalars for phenomenological purposes. One scenario, developed initially by Peccei and Quinn [39], proposes that a massive scalar field (an axion) provides a solution to the CP problem in QCD. Reference [37] discusses in detail whether the missing Peccei-Quinn axion can be coming from the C_4 -moduli. The present work might help answering this question also for the B_2, C_2 -axions. To study this, one should however also include open string moduli which was beyond the scope of this paper. Another possibility to use axions is for driving inflation in the early universe [40] (also called N-flation). In [38] and [13] the N-flation scenario with type IIB axions was considered and made plausible in some specific toy models. Therefore our work extends the possibility to study this idea as it provides a more systematic approach to the subject. We leave this for future research.

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A. Inverting the full Kähler metric

Here we will present in detail how to deal with inverting the matrix of partial derivatives $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$ after including the non-geometric moduli. We will not consider the complex structure moduli dependence, as they are not coupled to the dilaton and the other moduli in K :

$$K_{I\bar{J}} = \begin{pmatrix} K_{z^i \bar{z}^j} & 0 \\ 0 & K_{A\bar{B}} \end{pmatrix} \tag{A.1}$$

$K_{z^i \bar{z}^j}$ cannot be inverted explicitly without a given model, so we focus on inverting $K_{A\bar{B}}$. We will show that, although rather non-trivial, there is an exact analytic solution for the inverse metric $K^{A\bar{B}}$ both at tree level and with perturbative α' -corrections included. We will therefore consider these cases separately in different subsections.

A.1 Tree level

More explicitly, the relevant part of the Kähler potential (3.1) is

$$K = -\ln(-i(\tau - \bar{\tau})) - 2 \ln \left(\frac{(\frac{2}{3}(T_\alpha + \bar{T}_\alpha) - \frac{i}{2(\tau - \bar{\tau})} \kappa_{\alpha ab} (G - \bar{G})^a (G - \bar{G})^b) v^\alpha (T_\alpha, G^a, \tau)}{6} \right), \tag{A.2}$$

where v^α is an implicit function of the Kähler coordinates. It is given by the relation $\kappa_\alpha = \kappa_{\alpha\beta\gamma} v^\beta v^\gamma = \kappa_{\alpha\beta} v^\beta$, where we made the definition $\kappa_{\alpha\beta} \equiv \kappa_{\alpha\beta\gamma} v^\gamma$. Therefore, $v^\alpha = \kappa^{\alpha\beta} \kappa_\beta$. We make an analogous definition for the intersection numbers with Latin indices: $\kappa_{ab} \equiv \kappa_{\alpha ab} v^\alpha$.

Now we can calculate the actual Kähler metric, using the following matrix definitions that are used for shorthand and easier calculation:

$$G^{\alpha\beta} \equiv -\frac{2}{3} \kappa \kappa^{\alpha\beta} + 2v^\alpha v^\beta, \quad G_{ab} \equiv -\frac{3}{2} \frac{\kappa_{ab}}{\kappa}, \quad (\text{A.3})$$

and their corresponding inverses

$$G_{\alpha\beta} = -\frac{3}{2} \left(\frac{\kappa_{\alpha\beta}}{\kappa} - \frac{3}{2} \frac{\kappa_\alpha \kappa_\beta}{\kappa^2} \right), \quad G^{ab} = -\frac{2}{3} \kappa \kappa^{ab}. \quad (\text{A.4})$$

The first partial derivatives of K can then be computed to be:

$$\begin{aligned} K_\tau &= -K_{\bar{\tau}} = -\frac{1}{\tau - \bar{\tau}} - \frac{3i}{2(\tau - \bar{\tau})^2 \kappa} \kappa_{ab} (G - \bar{G})^a (G - \bar{G})^b \\ &= \frac{ie^\phi}{2} + iG_{ab} b^a b^b, \\ K_{G^a} &= -K_{\bar{G}^a} = \frac{3i}{(\tau - \bar{\tau}) \kappa} \kappa_{ab} (G - \bar{G})^b = 2iG_{ab} b^b, \\ K_{T_\alpha} &= K_{\bar{T}_\alpha} = -\frac{2v^\alpha}{\kappa}, \end{aligned} \quad (\text{A.5})$$

where we used from (2.6) that $(\tau - \bar{\tau}) = 2ie^{-\phi}$ and $(G - \bar{G})^a = -(\tau - \bar{\tau})b^a = -2ie^{-\phi}b^a$. Then,

$$\begin{aligned} K_{\tau\bar{\tau}} &= \frac{e^{2\phi}}{4} + e^\phi G_{ab} b^a b^b + \frac{9}{16\kappa^2} G^{\alpha\beta} \kappa_{\alpha ab} b^a b^b \kappa_{\beta cd} b^c b^d, \\ K_{G^a \bar{\tau}} &= K_{\tau \bar{G}^a} = e^\phi G_{ab} b^b + \frac{9}{8\kappa^2} G^{\alpha\beta} \kappa_{\alpha ab} b^b \kappa_{\beta cd} b^c b^d, \\ K_{T_\alpha \bar{\tau}} &= -K_{\tau \bar{T}_\alpha} = -\frac{3i}{4\kappa^2} G^{\alpha\beta} \kappa_{\beta ab} b^a b^b, \\ K_{G^a \bar{G}^b} &= e^\phi G_{ab} + \frac{9}{4\kappa^2} G^{\alpha\beta} \kappa_{\alpha ac} b^c \kappa_{\beta bd} b^d, \\ K_{T_\alpha \bar{G}^a} &= -K_{G^a \bar{T}_\alpha} = -\frac{3i}{2\kappa^2} G^{\alpha\beta} \kappa_{\beta ab} b^b, \\ K_{T_\alpha \bar{T}_\beta} &= \frac{G^{\alpha\beta}}{\kappa^2}. \end{aligned} \quad (\text{A.6})$$

The inverse metric can be found after a lengthy calculation, which goes as follows. One can make an ansatz for each of the elements of the inverse metric from the number of free indices, e.g. the component $K^{T_\alpha \bar{\tau}}$ has only one free lower index α as opposed to the upper index of the original metric component. Therefore, a possible ansatz could be $K^{T_\alpha \bar{\tau}} = a\kappa_\alpha + b\kappa_{\alpha ab} b^a b^b$, where a and b can be any expression with fully contracted indices (or with no indices at all). Plugging the ansatz for every element of the inverse matrix

leads to 9 coupled equations which lead to unique determination of all components. For the specific example, we find $a = 0$, $b = 3ie^{-\phi}$. Explicitly, the whole inverse metric is:

$$\begin{aligned}
 K^{\tau\bar{\tau}} &= 4e^{-2\phi}, \\
 K^{G^a\bar{\tau}} &= K^{\tau\bar{G}^a} = -4e^{-2\phi}b^a, \\
 K^{T_\alpha\bar{\tau}} &= -K^{\tau\bar{T}_\alpha} = 3ie^{-2\phi}\kappa_{\alpha ab}b^ab^b, \\
 K^{G^a\bar{G}^b} &= e^{-\phi}G^{ab} + 4e^{-2\phi}b^ab^b, \\
 K^{T_\alpha\bar{G}^a} &= -K^{G^a\bar{T}_\alpha} = -\frac{3ie^{-\phi}}{2}G^{ab}\kappa_{\alpha bc}b^c - 3ie^{-2\phi}\kappa_{\alpha bc}b^bb^cb^a, \\
 K^{T_\alpha\bar{T}_\beta} &= \kappa^2G_{\alpha\beta} + \frac{9e^{-\phi}}{4}G^{ab}\kappa_{aac}b^c\kappa_{\beta bd}b^d + \frac{9e^{-2\phi}}{4}\kappa_{\alpha ab}b^ab^b\kappa_{\beta cd}b^cb^d.
 \end{aligned} \tag{A.7}$$

Having found the inverse metric and the first partial derivatives (A.5), to obtain eq. (3.3) is down to some trivial algebra. However, it might be quite interesting in which way one gets the number 4. We can break up the sum into two parts - a sum which runs over all T_α and G^a but not over the dilaton, plus the remainder of the whole sum (i.e. where at least one of the indices goes over τ). Then,

$$K^{i\bar{j}}K_iK_{\bar{j}} = 3 + 9e^{-2\phi} \left(\frac{\kappa_{\alpha ab}v^\alpha b^ab^b}{\kappa} \right)^2, \quad i, j = T_1 \dots T_{h_+^{(1,1)}}, G^1 \dots G^{h_-^{(1,1)}}, \tag{A.8}$$

while for the remainder one gets $1 - 9e^{-2\phi} \left(\frac{\kappa_{\alpha ab}v^\alpha b^ab^b}{\kappa} \right)^2$ as expected since the two sums add up to 4.

A.2 Perturbative α' -corrections

In order to simplify notation, we first use the following definitions:

$$\begin{aligned}
 \hat{\xi} &\equiv \frac{\xi}{2(2i)^{3/2}}, \\
 Y &\equiv V_{\text{CY}} + \frac{\xi}{2} \left(\frac{\tau - \bar{\tau}}{2i} \right)^{3/2} = \frac{\kappa}{6} + \hat{\xi}(\tau - \bar{\tau})^{3/2}.
 \end{aligned} \tag{A.9}$$

In the following we will drop the hat of $\hat{\xi}$ and will use this new definition until the end of the section where we switch to the proper definition. With these identifications, the Kähler potential takes a misleadingly simple form:

$$K = -\ln(-i(\tau - \bar{\tau})) - \ln(Y). \tag{A.10}$$

However, Y is now dependent on all variables. Its partial derivatives are:

$$\begin{aligned}
 \frac{\partial Y}{\partial T_\alpha} &= \frac{\partial Y}{\partial \bar{T}_\alpha} = \frac{v^\alpha}{6} \\
 \frac{\partial Y}{\partial G^a} &= -\frac{\partial Y}{\partial \bar{G}^a} = -\frac{i}{4(\tau - \bar{\tau})}\kappa_{ab}(G - \bar{G})^b \\
 \frac{\partial Y}{\partial \tau} &= -\frac{\partial Y}{\partial \bar{\tau}} = \frac{i}{8(\tau - \bar{\tau})^2}\kappa_{ab}(G - \bar{G})^a(G - \bar{G})^b + \frac{3}{2}\xi(\tau - \bar{\tau})^{1/2},
 \end{aligned} \tag{A.11}$$

where we define $\kappa_{ab}, \kappa_{\alpha\beta}$ as in the previous subsection. However, we slightly change the definition of $G^{\alpha\beta}, G_{ab}$:

$$G^{\alpha\beta} \equiv -\frac{Y}{9}\kappa^{\alpha\beta} + \frac{1}{18}v^\alpha v^\beta, \quad G_{ab} \equiv -\frac{\kappa_{ab}}{4Y}. \quad (\text{A.12})$$

Their corresponding inverses are

$$G_{\alpha\beta} = -\frac{9\kappa_{\alpha\beta}}{Y} + \frac{\kappa_\alpha \kappa_\beta}{2Y(-\frac{Y}{9} + \frac{\kappa}{18})}, \quad G^{ab} = -4Y\kappa^{ab}. \quad (\text{A.13})$$

With these, and using $(\tau - \bar{\tau}) = 2ie^{-\phi}$, $(G - \bar{G})^a = -(\tau - \bar{\tau})b^a = -2ie^{-\phi}b^a$,

$$\begin{aligned} K_\tau = -K_{\bar{\tau}} &= -\frac{1}{\tau - \bar{\tau}} - \frac{i}{4(\tau - \bar{\tau})^2 Y} \kappa_{ab} (G - \bar{G})^a (G - \bar{G})^b - \\ & - \frac{3\xi(\tau - \bar{\tau})^{1/2}}{Y} = \frac{ie^\phi}{2} + iG_{ab}b^a b^b - \frac{3\xi(2i)^{1/2}e^{-\phi/2}}{Y}, \\ K_{G^a} = -K_{\bar{G}^a} &= \frac{i}{2(\tau - \bar{\tau})Y} \kappa_{ab} (G - \bar{G})^b = 2iG_{ab}b^b, \\ K_{T_\alpha} = K_{\bar{T}_\alpha} &= -\frac{v^\alpha}{3Y}. \end{aligned} \quad (\text{A.14})$$

The metric then takes the form:

$$\begin{aligned} K_{\tau\bar{\tau}} &= \left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \\ & + \left(e^\phi + \frac{3i\xi(2i)^{1/2}e^{-\phi/2}}{Y} \right) G_{ab}b^a b^b + \frac{9}{16Y^2} G^{\alpha\beta} \kappa_{\alpha ab} b^a b^b \kappa_{\beta cd} b^c b^d, \\ K_{G^a \bar{\tau}} = K_{\tau \bar{G}^a} &= \left(e^\phi + \frac{3i\xi(2i)^{1/2}e^{-\phi/2}}{Y} \right) G_{ab}b^b + \frac{9}{8Y^2} G^{\alpha\beta} \kappa_{\alpha ab} b^b \kappa_{\beta cd} b^c b^d, \\ K_{T_\alpha \bar{\tau}} = -K_{\tau \bar{T}_\alpha} &= -\frac{3i}{4Y^2} G^{\alpha\beta} \kappa_{\beta ab} b^a b^b - \frac{\xi(2i)^{1/2}e^{-\phi/2}}{2Y^2} v^\alpha, \\ K_{G^a \bar{G}^b} &= e^\phi G_{ab} + \frac{9}{4Y^2} G^{\alpha\beta} \kappa_{\alpha ac} b^c \kappa_{\beta bd} b^d, \\ K_{T_\alpha \bar{G}^a} = -K_{G^a \bar{T}_\alpha} &= -\frac{3i}{2Y^2} G^{\alpha\beta} \kappa_{\beta ab} b^b, \\ K_{T_\alpha \bar{T}_\beta} &= \frac{G^{\alpha\beta}}{Y^2}. \end{aligned} \quad (\text{A.15})$$

The inverse metric is found along the procedure from the previous subsection, described after eq. (A.6). For easier reading, we will write down the ansatz for the inverse metric

and give the resulting prefactors separately.

$$\begin{aligned}
K^{\tau\bar{\tau}} &= a, \\
K^{G^a\bar{\tau}} &= K^{\tau\bar{G}^a} = bb^a, \\
K^{T_{\alpha\bar{\tau}}} &= -K^{\tau\bar{T}_{\alpha}} = c\kappa_{\alpha ab}b^ab^b + d\kappa_{\alpha}, \\
K^{G^a\bar{G}^b} &= lG^{ab} + mb^ab^b, \\
K^{T_{\alpha}\bar{G}^a} &= -K^{G^a\bar{T}_{\alpha}} = eG^{ab}\kappa_{abc}b^cb^c + f\kappa_{abc}b^bb^cb^a + q\kappa_{\alpha}b^a, \\
K^{T_{\alpha}\bar{T}_{\beta}} &= gG_{\alpha\beta} + hG^{ab}\kappa_{aac}b^c\kappa_{\beta bd}b^d + j_1\kappa_{\alpha ab}b^ab^b\kappa_{\beta cd}b^cb^d + \\
&\quad + j_2(\kappa_{\alpha}\kappa_{\beta ab}b^ab^b + \kappa_{\alpha ab}b^ab^b\kappa_{\beta}) + j_4\kappa_{\alpha}\kappa_{\beta}.
\end{aligned} \tag{A.16}$$

The corresponding prefactors are

$$\begin{aligned}
a &= \frac{2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)}{2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}} \\
b &= -a \\
c &= \frac{3i\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)}{2\left(2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}\right)} \\
d &= -\frac{(2i)^{1/2}\xi e^{-\phi/2}}{2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}} \\
e &= -\frac{3i}{2}e^{-\phi} \\
f &= -c \\
q &= -d \\
g &= Y^2 \\
h &= \frac{9}{4}e^{-\phi} \\
j_1 &= \frac{9\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)}{8\left(2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}\right)} \\
j_2 &= \frac{3i\xi(2i)^{1/2}e^{-\phi/2}}{4\left(2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}\right)} \\
j_4 &= -\frac{2i\xi^2 e^{-\phi}}{\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(2\left(-\frac{Y}{9} + \frac{\kappa}{18}\right)\left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2}\right) + \frac{i\kappa\xi^2 e^{-\phi}}{Y^2}\right)} \\
l &= e^{-\phi} \\
m &= a.
\end{aligned} \tag{A.17}$$

Clearly the inverse metric in this form is not very suitable for calculational purposes. As we are interested in the large volume behavior, we can expand the coefficients a, \dots, m and take the leading terms in the limit where $V_{CY} \rightarrow \infty$. In order to calculate $K^{i\bar{j}}K_iK_{\bar{j}}$

exactly upto $\mathcal{O}(V_{\text{CY}}^{-5/3})$ we also need some of the subleading terms of the inverse metric. With this choice of relevant accuracy, we obtain:

$$\begin{aligned}
 K^{\tau\bar{\tau}} &\approx 4e^{-2\phi} - \frac{24\xi e^{-7\phi/2}}{(2i)^{1/2}V_{\text{CY}}}, \\
 K^{G^a\bar{\tau}} = K^{\tau\bar{G}^a} &\approx -4e^{-2\phi}b^a, \\
 K^{T_\alpha\bar{\tau}} = -K^{\tau\bar{T}_\alpha} &\approx 3ie^{-2\phi}\kappa_{\alpha ab}b^ab^b - \frac{9\xi(2i)^{1/2}e^{-5\phi/2}}{V_{\text{CY}}}\kappa_\alpha, \\
 K^{G^a\bar{G}^b} &\approx e^{-\phi}G^{ab} + 4e^{-2\phi}b^ab^b, \\
 K^{T_\alpha\bar{G}^a} = -K^{G^a\bar{T}_\alpha} &\approx -\frac{3ie^{-\phi}}{2}G^{ab}\kappa_{abc}b^c - 3ie^{-2\phi}\kappa_{abc}b^bb^cb^a + \frac{9\xi(2i)^{1/2}e^{-5\phi/2}}{V_{\text{CY}}}\kappa_\alpha b^a, \\
 K^{T_\alpha\bar{T}_\beta} &\approx Y^2G_{\alpha\beta} + \frac{9e^{-\phi}}{4}G^{ab}\kappa_{aac}b^c\kappa_{\beta bd}b^d + \frac{9e^{-2\phi}}{4}\kappa_{\alpha ab}b^ab^b\kappa_{\beta cd}b^cb^d + \\
 &\quad + \frac{27i\xi(2i)^{1/2}e^{-5\phi/2}}{4V_{\text{CY}}}(\kappa_\alpha\kappa_{\beta ab}b^ab^b + \kappa_{\alpha ab}b^ab^b\kappa_\beta) - \frac{81i\xi^2e^{-3\phi}}{V_{\text{CY}}^2}\kappa_\alpha\kappa_\beta.
 \end{aligned} \tag{A.18}$$

Now we can calculate $K^{i\bar{j}}K_iK_{\bar{j}}$ and we find it again equal to 4 as in the tree level case (3.3). This time the 4 comes as follows:

$$\begin{aligned}
 K^{i\bar{j}}K_iK_{\bar{j}} &= 3 + \frac{e^{-2\phi}|W|^2}{4V_{\text{CY}}^2}(\kappa_{\alpha ab}v^\alpha b^ab^b)^2 - \frac{6\hat{\xi}e^{-3\phi/2}}{(2i)^{1/2}V_{\text{CY}}} + \mathcal{O}(V_{\text{CY}}^{-5/3}), \\
 i, j &= T_1 \dots T_{h_+^{(1,1)}}, G^1 \dots G^{h_-^{(1,1)}},
 \end{aligned} \tag{A.19}$$

and the remainder is what is left such that the sum is 4. Note that we still have $\hat{\xi}$ dependence, and if we switch to ξ we recover the standard term that appears in the literature (c.f. (17) of [6]):

$$-\frac{6\hat{\xi}e^{-3\phi/2}}{(2i)^{1/2}V_{\text{CY}}} = \frac{3\xi e^{-3\phi/2}}{4V_{\text{CY}}}.$$

B. Proof for the existence of minima of the moduli potential in the b^a -directions

Here we present an extensive argument to show that the moduli potential V in its form (5.7) will always exhibit at least one minimum with respect to the moduli in question, κ_s, V_{CY} , and $h_-^{1,1} b^a$'s. The argument can be trivially extended for the case of many small 4-cycles κ_{s_i} (c.f. eq. (5.13)).

Apart from the coefficients α, β, γ , we see that the potential V (5.7) depends on b^a only through $\kappa_{sab}b^ab^b$. Therefore, let us define $x \equiv \kappa_{sab}b^ab^b$ and take V as a function of only $x, \kappa_s, V_{\text{CY}}$. We will then consider all possible cases of scaling of β and γ with x (α does not really depend on x since any change in the b^a 's only changes the value of ρ_s at its minimum and leaves α the same). So,

$$V(x, \kappa_s, V_{\text{CY}}) = -\alpha \frac{\kappa_s e^{-\kappa_s} e^x}{V_{\text{CY}}^2} + \frac{\beta(x)}{V_{\text{CY}}^3} + \frac{\gamma(x)\sqrt{\kappa_s} e^{-2\kappa_s} e^{2x}}{V_{\text{CY}}}, \tag{B.1}$$

where we implicitly absorbed additional constants in the definitions of $\alpha, \beta, \gamma, \kappa_s, x$ in order to simplify notation and without any loss of generality. Note that here α, β, γ are always strictly positive, while $x \in (-\infty, +\infty)$ (for a completely generic matrix κ_{sab}) and $\kappa_s, V_{CY} \in (0, +\infty)$. Since $V : \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+ \mapsto \mathbb{R}$ we cannot really picture it, but we will present three slices of V at different values of x : 0 and $\pm\infty$. Then we will be able to draw conclusions on how the potential looks everywhere.

- $x \rightarrow -\infty$:

$$\lim_{x \rightarrow -\infty} V = \frac{\beta(x \rightarrow -\infty)}{V_{CY}^3}.$$

Since $\beta(x \rightarrow -\infty)$ is always positive (in this limit it is in fact going to positive infinity), we see that for all values of κ_s, V_{CY} the potential remains positive and vanishes from above when $V_{CY} \rightarrow \infty$.

- $x = 0$:

$$V = -\alpha_{LVS} \frac{\kappa_s e^{-\kappa_s}}{V_{CY}^2} + \frac{\beta_{LVS}}{V_{CY}^3} + \frac{\gamma_{LVS} \sqrt{\kappa_s} e^{-2\kappa_s}}{V_{CY}}.$$

Here the standard LVS is reproduced, the potential V is large positive for small values of κ_s, V_{CY} , then goes below zero as they increase, and approaches zero asymptotically from below as $V_{CY} \rightarrow \infty$. The minimum of the potential is at $\kappa_s \approx \ln(V_{CY})$ and some finite value of V_{CY} that depends on the coefficients and is not relevant for the argument here.

- $x \rightarrow \infty$:

$$\lim_{x \rightarrow \infty} V = -\alpha \frac{\kappa_s e^{-\kappa_s} e^\infty}{V_{CY}^2} + \frac{\beta(x \rightarrow \infty)}{V_{CY}^3} + \frac{\gamma(x \rightarrow \infty) \sqrt{\kappa_s} e^{-2\kappa_s} e^{2\infty}}{V_{CY}}.$$

This case is particularly subtle and we need to split it into a few subcases.

For finite κ_s we see that the last term is largely dominant as it rises squarely faster than the first term. This makes the potential always positive for finite values of the volume. When $V_{CY} \rightarrow 0$ the second term will make sure the potential never goes negative.

On the other hand, when κ_s goes faster to ∞ than x the second term will dominate everywhere and the potential is positive and only vanishing as $V_{CY} \rightarrow \infty$.

The most subtle case is when κ_s goes to infinity together with x . Then, $e^{x-\kappa_s}$ will remain finite and

$$\lim_{\kappa_s \rightarrow x, x \rightarrow \infty} V = \lim_{x \rightarrow \infty} -\alpha \frac{x}{V_{CY}^2} + \frac{\beta(x)}{V_{CY}^3} + \frac{\gamma(x) \sqrt{x}}{V_{CY}}.$$

Consider $\beta = const. + \frac{(\kappa_{ab} b^a b^b)^2}{V_{CY}}$. $x \rightarrow \infty$ only when at least one of the b^a 's goes to infinity. But then, since κ_{ab} is negative definite, $(\kappa_{ab} b^a b^b)^2$ will necessarily also become infinite. Therefore, in the limit $x \rightarrow \infty$ the second term will always scale as $\frac{x^2}{V_{CY}^{10/3}}$ (remember that $\kappa_{ab} \equiv \kappa_{\alpha ab} v^\alpha$). The scaling of $\gamma(x)$ is less clear, but this is not

important for our argument. We can even neglect the γ term completely (since in any case it gives a positive contribution) and still prove our point. The potential is then simply $-\frac{\alpha'}{V_{\text{CY}}^2} + \frac{\beta'}{V_{\text{CY}}^{10/3}}$ with the x -dependence hidden in $\alpha' \sim x, \beta' \sim x^2$. Then it is straightforward to minimize the potential in the volume direction, and the value of the potential at the minimum is

$$V_{\text{min}} = -\frac{6\sqrt{3}\alpha'^{5/2}}{25\sqrt{5}\beta'^{3/2}} \sim -\frac{1}{\sqrt{x}} \Rightarrow \lim_{x \rightarrow \infty} V_{\text{min}} = 0.$$

The minimum of the potential increases with x , so although V can be (infinitesimally) negative, its real minimum will not be at $x \rightarrow \infty$ but at some finite value of x .

We have exhausted the limiting cases and showed there is no runaway direction for x and it must remain finite in order to minimize V . And for finite fixed $x = x_0$ the potential will have no runaway directions in the κ_s and V_{CY} directions. This is the case because the standard LVS ($x_0 = 0$) behavior of the potential will still hold, only that for $x_0 \neq 0$ the relative weight between the coefficients $\alpha_{\text{eff}} \equiv \alpha e^{x_0}, \beta_{\text{eff}} = \beta(x_0), \gamma_{\text{eff}} = \gamma(x_0)e^{2x_0}$ will change, effectively changing the values of κ_s, V_{CY} at the corresponding AdS minimum. Since β_{eff} always remains positive, κ_s, V_{CY} will be finite at the minimum and the minimum itself will be at a finite value V_{min} (c.f. appendix A of [33] for a detailed proof).

Now we can safely claim that in all cases the full potential $V(x, \kappa_s, V_{\text{CY}})$ will be minimized at a point or points inside the domain of the variables, i.e. $x, \kappa_s, V_{\text{CY}}$ will all have finite values at the minima. We are unable to specify the number of minima, but we know there will be at least one since the minimum of the potential is finite and negative at the LVS slice $x = 0, \kappa_s \approx \ln(V_{\text{CY}})$, while on the boundaries of its domain it is positive or vanishing. This concludes our proof for the existence of minima of (5.7).

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